Abstract: A rational function $f(z)$ with complex coefficients defines a holomorphic map from the Riemann sphere to itself. Some aspects of the global dynamical behavior of $f$ can be predicted from the orbits, under $f$, of the critical points of $f$ (i.e. points at which the derivative of $f$ vanishes). If every critical point of $f$ has a finite orbit, then $f$ is called post-critically finite (PCF).

Suppose $\phi$ is a PCF branched covering from a topological two-sphere to itself. One can ask: is $\phi$ homotopic to a PCF rational function from the Riemann sphere to itself? Thurston answered this question by producing a holomorphic dynamical system $T(\phi)$ induced by $\phi$ on the Teichmüller space of complex structures on the topological sphere. Koch found that $T(\phi)$ descends to an algebraic dynamical system $H(\phi)$ on the moduli space of configurations of points on the Riemann sphere.

I will introduce 3(+) interconnected dynamical systems: topological ($\phi$), holomorphic ($T(\phi)$) and algebraic ($H(\phi)$).