Round 1

12 PROBLEMS + ANSWERS

1. A sequence begins 10, 11. Each later term is found by subtracting 1 from the sum of the previous two terms. Find the seventh term.
2. How many distinct strings can be made by rearranging the letters A,A,B,C?
3. Compute $52 \cdot 10 - 14$.
4. Find the remainder when $x^2 + 4x + 7$ is divided by $x - 2$.
5. A circle of radius 3 is centered at $(0, 1)$. Find the point on the circle closest to $(5, 13)$.
6. In radians, approximate $\cos(1/5)$ to the nearest hundredth.
7. How many permutations of 1, 2, 3, 4 have 1 before 2?
8. What is the remainder when 12893 is divided by 9?
9. A quadratic function f satisfies $f(0) = 3$, $f(1) = 6$, and $f(2) = 15$. Find $f(3)$.
10. The average of 5 numbers is 11. What is the average after the additional number 17 is included?
11. Among all rectangles with perimeter 40, what is the greatest possible area?
12. Three ordinary fair dice are rolled. What is the probability that their sum is 8?

Answers

1. 126
2. 12
3. 506
4. 19
5. $\left(\frac{15}{13}, \frac{49}{13}\right)$
6. 0.98
7. 12
8. 5
9. 30
10. 12
11. 100
12. $\frac{7}{72}$

Round 2

12 PROBLEMS + ANSWERS

1. The numbers 1 through 7 are placed in a bucket. If 5 numbers are drawn without replacement, what is the probability that the first number drawn is the smallest of those drawn?
2. A quadratic function f satisfies $f(0) = 6$, $f(1) = 7$, and $f(2) = 14$. Find $f(3)$.
3. What is 25% of 100?
4. What is the sum of the interior angles of a pentagon?
5. To the nearest tenth, approximate $\sqrt{104}$.
6. What is the sum of the squares of the distinct prime divisors of 6?
7. Let $f(x) = x + \frac{1}{x}$. Find $f(\sqrt{5})$.
8. In GPT, what does the letter T stand for?
9. Alex can complete one job in 8 hours and Blair can complete one job in 16 hours. How many hours will they need to complete 3 jobs while working together?
10. How many permutations of $1, 2, \ldots, 6$ have 1 before 2, 3 before 4, and 5 before 6?
11. Find the remainder when $x^3 + 4x^2 + 5x + 3$ is divided by $x - 1$.
12. What is the sum of the squares of the first 5 positive integers?

Answers

1. $\frac{1}{5}$
2. 27
3. 25
4. 540°
5. 10.2
6. 13
7. $\frac{6\sqrt{5}}{5}$
8. Transformer
9. 16 hours
10. 90
11. 13
12. 55

Round 3

12 PROBLEMS + ANSWERS

1. Evaluate the infinite geometric series $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$.
2. Find an equation of the line through $(0, 7)$ that is perpendicular to $y = 2x$.
3. In AGI, what does the letter G stand for?
4. How many distinct strings can be made by rearranging the letters A,A,A,B,B,C,C?
5. A random permutation of 1, 2, 3, 4, 5, 6 is chosen. What is the probability that its first entry is the largest entry?
6. How many prime numbers are strictly between 20 and 40?
7. A cubic function f has values $0, 3, 10, 27$ at $x = 0, 1, 2, 3$, respectively. Find $f(4)$.
8. What is 35% of 80?
9. A triangle has side lengths 4, 5, and 7. What is the cosine of its largest angle?
10. Alex can complete one job in 5 hours and Blair can complete one job in 10 hours. How many hours will they need to complete 3 jobs while working together?
11. Compute $54 \cdot 11 - 17$.
12. What is the sum of the squares of the distinct prime divisors of 675?

Answers

1. $\frac{1}{2}$
2. $y = -\frac{1}{2}x + 7$
3. General
4. 210
5. $\frac{1}{6}$
6. 4
7. 60
8. 28
9. $-\frac{1}{5}$
10. 10 hours
11. 577
12. 34

Round 4

12 PROBLEMS + ANSWERS

1. Express the decimal number 255 in base 2.
2. How many integers from 1 through 120 are divisible by 4 or 9 or both?
3. A quadratic function f satisfies $f(0) = 2$, $f(1) = 3$, and $f(2) = 8$. Find $f(3)$.
4. Find $\sin(\arccos(8/10))$.
5. How many lattice paths go from $(0, 0)$ to $(4, 2)$ using only steps one unit right or one unit up?
6. Three ordinary fair dice are rolled. What is the probability that their sum is 11?
7. A right triangle has legs of length 15 and 20. What is the length of its hypotenuse?
8. Express 2026 as the sum of two four-digit primes.
9. The average of 8 numbers is 9. What is the average after the additional number 18 is included?
10. What is 15% of 60?
11. What is the remainder when 13030 is divided by 10?
12. A sequence begins with 2, and each later term is twice the previous term plus 1. Find the fifth term.

Answers

1. 11111111_2
2. 40
3. 17
4. $\frac{3}{5}$
5. 15
6. $\frac{1}{8}$
7. 25
8. $1013 + 1013$
9. 10
10. 9
11. 0
12. 47

Round 5

12 PROBLEMS + ANSWERS

1. Find the remainder when $x^3 + 2x^2 + 7x + 5$ is divided by $x + 2$.
2. How many lattice paths go from $(0, 0)$ to $(5, 2)$ using only steps one unit right or one unit up?
3. In radians, approximate $\sin(0.1234)$ to the nearest hundredth.
4. A point is chosen uniformly from the square $[0, 2] \times [0, 2]$. What is the probability that its distance from the origin is at most 2?
5. Compute $48 \cdot 8 - 8$.
6. The average of 9 numbers is 10. What is the average after the additional number 30 is included?
7. A cubic function has values 0, 2, 12, and 36 at $x = 0, 1, 2, 3$, respectively. Find its value at $x = 4$.
8. Find $\tan(\arcsin(8/17))$.
9. What is the remainder when 12619 is divided by 7?
10. What is the volume of a sphere of radius 3?
11. A sequence begins 1, 2. Each later term is one more than the sum of the previous two terms. Find the eighth term.
12. Alex can complete one job in 4 hours and Blair can complete one job in 8 hours. How many hours will they need to complete 3 jobs while working together?

Answers

1. -9
2. 21
3. 0.12
4. $\frac{\pi}{4}$
5. 376
6. 12
7. 80
8. $\frac{8}{15}$
9. 5
10. 36π
11. 54
12. 8 hours

Round 6

12 PROBLEMS + ANSWERS

1. How many lattice paths go from $(0, 0)$ to $(2, 3)$ using only steps one unit right or one unit up?
2. Compute $56 \cdot 7 - 20$.
3. What is the sum of the squares of the distinct prime divisors of 1372?
4. An isosceles triangle has side lengths 5, 5, and 6. Find the tangent of either smallest angle.
5. Three ordinary fair dice are rolled. What is the probability that their sum is 6?
6. What is the positive difference between the roots of $x^2 - 8x + 15 = 0$?
7. One worker completes a job in 4 hours and another completes it in 6 hours. How many hours do they need to complete one job together?
8. A sequence begins 2, 3. Each later term is found by taking the sum of the previous two terms. Find the seventh term.
9. If $a + b = 8$ and $ab = 15$, find $a^2 + b^2$.
10. Find the area of a triangle with side lengths 5, 5, and 6.
11. What is the remainder when 2^{10} is divided by 7?
12. Express the decimal number 169 in base 2.

Answers

1. 10
2. 372
3. 53
4. $\frac{4}{3}$
5. $\frac{5}{108}$
6. 2
7. $\frac{12}{5}$ hours
8. 34
9. 34
10. 12
11. 2
12. 10101001_2

Round 7

12 PROBLEMS + ANSWERS

1. How many distinct strings can be made by rearranging the letters A,A,A,B,B,C?
2. What is the sum of the distinct prime divisors of 35?
3. The average of 9 numbers is 15. What is the average after the additional number 25 is included?
4. What is 30% of 120?
5. An ordinary die and a die labeled 2, 3, 5, 7, 11, 13 are rolled. What is the probability that the two results have sum greater than 10?
6. Find an equation of the line through $(3, 5)$ that is perpendicular to $y = 4x$.
7. Compute $2^3 \cdot 5^5$.
8. Find the remainder when $x^3 + 2x^2 + 3x + 1$ is divided by $x + 1$.
9. A sequence begins with 1, and each later term is three times the preceding term minus 1. Find the fourth term.
10. The length of a rectangle is 3 times its width. If its perimeter is 32, what is its area?
11. A quadratic function has values 2, 6, and 12 at $x = 0, 1, 2$, respectively. Find its value at $x = 3$.
12. Today is Friday. What day of the week will it be 274 days from today?

Answers

1. 60
2. 12
3. 16
4. 36
5. $\frac{4}{9}$
6. $y = -\frac{1}{4}x + \frac{23}{4}$
7. 25000
8. -1
9. 14
10. 48
11. 20
12. Saturday

Round 8

12 PROBLEMS + ANSWERS

1. What is the volume of a right circular cylinder with radius 4 and height 5?
2. If $a + b = 22$ and $ab = 120$, find $a^2 + b^2$.
3. Evaluate the infinite geometric series $5 + 1 + \frac{1}{5} + \frac{1}{25} + \cdots$.
4. Today is Thursday. What day of the week will it be 385 days from today?
5. What is the remainder when 12756 is divided by 8?
6. Two ordinary fair dice are rolled. What is the probability that their sum is 8?
7. The length of a rectangle is 4 times its width. If its perimeter is 80, what is its area?
8. How many lattice paths from $(0, 0)$ to $(5, 6)$ use only right and up steps and pass through $(2, 3)$?
9. What is the sum of the squares of the first 4 positive integers?
10. Express the decimal number 118 in base 2.
11. Find the remainder when $x^3 + x^2 + 4x + 8$ is divided by $x - 1$.
12. Find $\cos(\arctan(3/4))$.

Answers

1. 80π
2. 244
3. $\frac{25}{4}$
4. Thursday
5. 4
6. $\frac{5}{36}$
7. 256
8. 200
9. 30
10. 1110110_2
11. 14
12. $\frac{4}{5}$

Round 9

12 PROBLEMS + ANSWERS

1. Express 2026 as a sum of two positive perfect squares.
2. The average of 7 numbers is 8. What is the average after the additional number 24 is included?
3. How many lattice paths go from $(0, 0)$ to $(3, 3)$ using only steps one unit right or one unit up?
4. A sequence begins 5, 6. Each later term is found by taking the sum of the previous two terms. Find the seventh term.
5. A quadratic function f satisfies $f(0) = 4$, $f(1) = 6$, and $f(2) = 10$. Find $f(3)$.
6. What is 10% of 140?
7. Given $\ln 2 \approx 0.6931$ and $\ln 3 \approx 1.0986$, approximate $\ln 144$ to the nearest hundreth.
8. The numbers 1 through 8 are placed in a bucket. If 6 numbers are drawn without replacement, what is the probability that the first number drawn is the smallest of those drawn?
9. Find the remainder when $x^3 + 4x^2 + 3x + 7$ is divided by x.
10. What is the circumference of a circle of radius 7?
11. If $a + b = 6$ and $ab = 8$, find $a^2 + b^2$.
12. Express the decimal number 84 in base 3.

Answers

1. $45^2 + 1^2$
2. 10
3. 20
4. 73
5. 16
6. 14
7. 4.97
8. $\frac{1}{6}$
9. 7
10. 14π
11. 20
12. 10010_3

Round 10

12 PROBLEMS + ANSWERS

1. How many lattice paths go from $(0, 0)$ to $(3, 2)$ using only steps one unit right or one unit up?
2. In what decade did Alan Turing help break the Enigma code?
3. Find $\sin(\arccos(12/13))$.
4. Find an equation of the line through $(3, 3)$ that is perpendicular to $y = x$.
5. A sequence begins 6, 7. Each later term is found by adding 1 to the sum of the previous two terms. Find the seventh term.
6. What is the sum of the distinct prime divisors of 108?
7. The numbers 1 through 11 are placed in a bucket. If 5 numbers are drawn without replacement, what is the probability that the first number drawn is the smallest of those drawn?
8. Express the decimal number 100 in base 4.
9. A triangle has side lengths 5, 6, and 8. What is the cosine of its largest angle?
10. How many integers from 1 through 60 are divisible by 4 or 6 or both?
11. What is 20% of 80?
12. What is the sum of all real solutions of $|2x - 5| = 9$?

Answers

1. 10
2. The 1940s
3. $\frac{5}{13}$
4. $y = -x + 6$
5. 98
6. 5
7. $\frac{1}{5}$
8. 1210_4
9. $-\frac{1}{20}$
10. 20
11. 16
12. 5

Round 11

12 PROBLEMS + ANSWERS

1. Today is Sunday. What day of the week will it be 311 days from today?
2. In artificial intelligence, what does LLM stand for?
3. Find an equation of the line through $(4, 6)$ that is perpendicular to $y = x$.
4. By what percentage does the area of a circle decrease when its radius is halved?
5. What is the remainder when 12482 is divided by 6?
6. A sequence begins 3, 4. Each later term is found by adding 1 to the sum of the previous two terms. Find the seventh term.
7. Solve for x in $\sqrt{x+7} - \sqrt{x} = 1$.
8. What is the volume of a right circular cylinder with radius 2 and height 4?
9. How many distinct strings can be made by rearranging the letters A,A,B,B,C,C?
10. The numbers 1 through 10 are placed in a bucket. If 4 numbers are drawn without replacement, what is the probability that the first number drawn is the smallest of those drawn?
11. Express the decimal number 67 in base 2.
12. What is the remainder when 2026 is divided by 7?

Answers

1. Wednesday
2. Large Language Model
3. $y = -x + 10$
4. 75%
5. 2
6. 59
7. 9
8. 16π
9. 90
10. $\frac{1}{4}$
11. 1000011_2
12. 3

Round 12

12 PROBLEMS + ANSWERS

1. A sequence begins 4, 5. Each later term is found by subtracting 1 from the sum of the previous two terms. Find the seventh term.
2. Find $\cos(\arctan(7/24))$.
3. In what decade was the Navajo Code used during World War II?
4. How many integers from 1 through 100 are divisible by 3 or 7 or both?
5. How many lattice paths go from $(0, 0)$ to $(4, 4)$ using only steps one unit right or one unit up?
6. Let $f(x) = x^2 + 1$ and $g(x) = 2x - 1$. Find $f(g(3))$.
7. The length of a rectangle is 4 times its width. If its perimeter is 50, what is its area?
8. Find the remainder when $x^3 + x^2 + 6x + 4$ is divided by $x - 2$.
9. Two distinct numbers are chosen uniformly from $\{1, 2, 3, 4, 5\}$. What is the probability that their sum is even?
10. What is $\sqrt[3]{1331}$?
11. If $a + b = 10$ and $ab = 24$, find $a^2 + b^2$.
12. The average of 6 numbers is 7. What is the average after the additional number 14 is included?

Answers

1. 48
2. $\frac{24}{25}$
3. The 1940s
4. 43
5. 70
6. 26
7. 100
8. 28
9. $\frac{2}{5}$
10. 11
11. 52
12. 8

Round 13

12 PROBLEMS + ANSWERS

1. To the nearest tenth, approximate $\sqrt{408}$.
2. What is the sum of the decimal digits of 2026?
3. Find an equation of the line through $(1, 3)$ that is perpendicular to $y = 2x$.
4. The length of a rectangle is 2 times its width. If its perimeter is 36, what is its area?
5. What is the smallest positive integer that leaves remainder 4 when divided by 5 and remainder 3 when divided by 7?
6. How many diagonals does a regular hexagon have?
7. Today is Wednesday. What day of the week will it be 237 days from today?
8. The numbers 1 through 9 are placed in a bucket. If 3 numbers are drawn without replacement, what is the probability that the first number drawn is the smallest of those drawn?
9. If $a + b = 14$ and $ab = 48$, find $a^2 + b^2$.
10. A cube has volume 64. What is its surface area?
11. What is the sum of the distinct prime divisors of 3267?
12. Express the decimal number 101 in base 5.

Answers

1. 20.2
2. 10
3. $y = -\frac{1}{2}x + \frac{7}{2}$
4. 72
5. 24
6. 9
7. Tuesday
8. $\frac{1}{3}$
9. 100
10. 96
11. 14
12. 401_5

Round 14

12 PROBLEMS + ANSWERS

1. Compute $2^2 \cdot 5^6$.
2. Alex can complete one job in 7 hours and Blair can complete one job in 14 hours. How many hours will they need to complete 3 jobs while working together?
3. How many distinct strings can be made by rearranging the letters A,A,A,B,C?
4. If $a + b = 12$ and $ab = 35$, find $a^2 + b^2$.
5. Find $\tan(\arccos(5/13))$.
6. Find the remainder when $x^3 + 3x^2 + 4x + 2$ is divided by x.
7. How many distinct arrangements can be made from the letters in MATH?
8. A fair 7-sided die is rolled, giving N. A fair coin is then flipped $6N$ times. What is the expected number of heads?
9. Express the decimal number 135 in base 3.
10. Today is Wednesday. What day of the week will it be 496 days from today?
11. A cube has volume 216. What is its surface area?
12. What is the median of the first 11 prime numbers?

Answers

1. 62500
2. 14 hours
3. 20
4. 74
5. $\frac{12}{5}$
6. 2
7. 24
8. 12
9. 12000_3
10. Tuesday
11. 216
12. 13

Round 15

12 PROBLEMS + ANSWERS

1. What is the mean of the first six prime numbers?
2. The numbers 1 through 6 are placed in a bucket. If 4 numbers are drawn without replacement, what is the probability that the first number drawn is the smallest of those drawn?
3. How many distinct strings can be made by rearranging the letters A,A,A,A,B,B,B,C?
4. Today is Tuesday. What day of the week will it be 348 days from today?
5. What is the smallest positive integer that leaves remainder 3 when divided by 8 and remainder 7 when divided by 11?
6. What is 5!?
7. If $\sin\theta = \frac{5}{13}$ and θ is acute, find $\sin(2\theta)$.
8. A right triangle has legs of length 6 and 8. What is the length of its hypotenuse?
9. To the nearest hundredth, approximate $\sqrt{10006}$.
10. Find an equation of the line through $(2, 4)$ that is perpendicular to $y = 3x$.
11. Find the remainder when $x^4 + 1$ is divided by $x - 2$.
12. A car travels one distance at 40 miles per hour and returns over the same distance at 60 miles per hour. What is its average speed for the whole trip?

Answers

1. $\frac{41}{6}$
2. $\frac{1}{4}$
3. 280
4. Sunday
5. 51
6. 120
7. $\frac{120}{169}$
8. 10
9. 100.03
10. $y = -\frac{1}{3}x + \frac{14}{3}$
11. 17
12. 48 miles per hour

Round 16

12 PROBLEMS + ANSWERS

1. The length of a rectangle is 3 times its width. If its perimeter is 56, what is its area?
2. What is the expected value of a roll of an ordinary fair six-sided die?
3. How many positive integers at most 20 are relatively prime to 20?
4. A sequence begins 9, 10. Each later term is found by adding 1 to the sum of the previous two terms. Find the seventh term.
5. Express the decimal number 2026 in base 5.
6. A quadratic function f satisfies $f(0) = 5$, $f(1) = 9$, and $f(2) = 17$. Find $f(3)$.
7. What is a positive integer called when it equals the sum of its positive divisors smaller than itself?
8. Let $f(x) = 3x - 2$ and $g(x) = x^2$. Find $g(f(2))$.
9. Alex can complete one job in 6 hours and Blair can complete one job in 12 hours. How many hours will they need to complete 3 jobs while working together?
10. Compute $50 \cdot 9 - 11$.
11. What is the volume of a right circular cone with radius 3 and height 4?
12. How many distinct strings can be made by rearranging the letters A,A,B,B,C?

Answers

1. 147
2. $\frac{7}{2}$
3. 8
4. 137
5. 31101_5
6. 29
7. A perfect number
8. 16
9. 12 hours
10. 439
11. 12π
12. 30