Mathematics Colloquium

Jake Levinson

Simon Fraser University

Tuesday, April 18, 2023
Time: 4:00 p.m.
Location: Weber 223

Title: Enumerative geometry and the Shapiro--Shapiro Conjecture

Abstract: In three dimensions, how many lines pass through four given lines? This question comes from classical complex enumerative geometry (Schubert calculus); nowadays, it and many similar problems are well understood, in part due to the theory of moduli spaces. Over the real numbers, however, most such problems do not have satisfactory answers -- in general there is no reason for solutions to be real.

The Shapiro--Shapiro conjecture, made in the 1990s and proven by Mukhin--Tarasov--Varchenko in 2009, is a striking exception: a class of problems in real Schubert calculus whose solutions turn out, always, to be real. The conjecture comes from the geometry of curves and has since found applications in combinatorics, representation theory and other parts of algebraic geometry.

I will discuss this problem and some joint work with Kevin Purbhoo that revisits it from a topological perspective, while allowing certain parameters to be complex. We identify the topological degree of the problem as a symmetric group character, which gives a lower bound for the number of real solutions. In the case where the parameters are all real, this gives a new proof of the original Shapiro--Shapiro conjecture.

MATHEMATICS
COLORADO STATE UNIVERSITY