Persistent Homology of Products and Gromov-Hausdorff Distances Between Hypercubes and Spheres

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Abstract

An exploration in the first half of this dissertation of the relationships among spectral sequences, persistent homology, and products of simplices, including the development of a new concept in categorical product filtration, is followed in the second half by new determinations of a) lower bounds for the Gromov-Hausdorff distance between n-spheres and $(n+1)$-hypercubes equipped with the geodesic metric and of b) new lower bounds for the coindexes of the Vietoris-Rips complexes of hypercubes equipped with the Hamming metric. In their paper,"Spectral Sequences, Exact Couples, and Persistent Homology of Filtrations" [**?**], Basu and Parida worked on building an n-derived exact couple from an increasing filtration X of simplicial complexes, $\mathcal{C}^{(n)}(X) = \{D^{(n)}(X), E^{(n)}(X), i^{(n)}, j^{(n)}, \partial^{(n)}\}$. The terms $E^{(n)}_{*,*}(X)$ are the bigraded vector spaces of a spectral sequence that has differentials $d^{(r)}(X)$, and the terms $D^{(n)}_{*,*}(X)$ are the persistent homology groups $H^{*,*}_{*}(X)$. They proved that there exists a long exact sequence whose groups are $H^{*,*}_{*}(X)$ and whose bigraded vector spaces are $(E^{*}_{*,*}(X), d^{*}(X))$. We establish in Section **??** of this dissertation a new, similar

theorem in the case of the categorical product filtration $X \times Y$ that states that there exists a long exact sequence consisting of $\bigoplus_{l+j=n} H_l^{*,*}(X) \otimes H_j^{*,*}(Y)$ and of the bigraded vector spaces $E^*_{*,*}(X \times Y)$ of $(E^*_{*,*}(X \times Y), d^*(X \times Y))$, and prove it in part using Künneth formulas on homology. The emphasis on product spaces continues in Section **??**, where we establish new lower bounds for the Gromov-Hausdorff distance between n-spheres and $(n+1)$-hypercubes, I^{n+1}, when both are equipped with the geodesic distance. From these lower bounds, we conjecture new lower bounds for the coindices of the Vietoris-Rips complexes of hypercubes when equipped with the Hamming metric. We then determine new lower bounds for the coindices of the Vietoris-Rips complexes of hypercubes, a) by producing a map between spheres and the geometric realizations of Vietoris-Rips complexes of hypercubes using abstract convex combination and balanced sets, and b) by decomposing hollow n-cubes (homotopically equivalent to the above-mentioned spheres) into simplices of smaller dimension and smaller diameter.