CONTENTS:
For each of the 8 QS courses (501, 517, 519, 545, 560, 566, 570, 574):
This document contains: recommended topics, optional topics, and recommended textbooks.
We are hoping to vote on these by the end of the semester.

Math 501 - Combinatorics I

M501 assumed knowledge:
Mathematical reasoning and proof techniques, undergraduate level abstract algebra, binomial coefficients and Pascal's triangle.

Recommended topics:
Counting Methods: enumeration, bijective proofs, inclusion-exclusion, pigeonhole.
Generating functions and recursion.
Symmetric functions.
Equivalence and Order Relations, Partitions, Lattices.
Incidence Structures: Graphs, Geometries.

Optional topics:
Logic and set theory.
Automata & Algorithms.
Structures in projective planes.
Strongly regular graphs.
Structural theory of graphs (isomorphism and reconstruction).
Polya theory.
Coding theory.
Moebius inversion.
Game theory / Voting Theory.
Combinatorial species.

Recommended Texts:
Combinatorics by Cameron
Enumerative Combinatorics, volumes I and II, by Stanley
The art of counting by Sagan
Combinatorial Methods by Gross

Math 517 - Introduction to real analysis

M517 assumed knowledge:
Single and multivariable calculus, mathematical reasoning and proof techniques, elementary linear algebra, elementary differential equations.

Recommended topics:
Metrics and overview of construction of real numbers
Sequences, convergence, Cauchy sequences of numbers
Completeness, open sets, continuity, compactness
The idea of infinite dimensional spaces, examples, norms
Sets of functions, norms on functions
Sequences of functions, convergence (including pointwise, uniform)
Metric spaces, completeness
Banach fixed point theorem, implicit and inverse function theorems
Compactness of infinite-dimensional spaces
Basics of Lebesgue integration theory

Optional topics:
Introduction to functional analysis and PDEs

Recommended books:
A Passage to Modern Analysis, Terrell (advanced undergrad to graduate)
Principles of Mathematical Analysis, Rudin (advanced undergrad to graduate)
The Elements of Real Analysis, Bartle (advanced undergrad to graduate)
Mathematical Analysis, Apostol

Math 519 - Complex Variables I

M519 assumed knowledge: real analysis at the undergraduate level.

Recommended Topics:

Analyticity of functions on the complex plane; Cauchy-Riemann conditions. The d-bar operator.
Examples of analytic functions: exponentials, trigonometric functions, logarithm
Branch cuts and elementary Riemann surfaces (nth roots and logs)
Power series, radius of convergence, identity theorem, analytic continuation
Integrals along curves, local primitives
Cauchy's theorem and applications, Morera's theorem
Cauchy integral formula and applications (integral formula for derivatives, Poisson kernel)
Cauchy inequalities, Liouville theorem and Fundamental Theorem of Algebra
Maximum Modulus principle and applications
Classification of isolated singularities
Laurent series
Residue Calculus
Argument Principle & applications (Rouché's theorem, Hurwitz's theorem, Open mapping theorem)
Conformal mappings and Linear fractional transformations

Optional Topics:
The optional topics may vary widely depending on the viewpoint of the instructor.

Additional tools in complex analysis; for example:
Generalized Cauchy integral formula, Plemelj formulas
Infinite products, Hadamard factorization theorem
Maps from the disk to itself, Schwarz lemma, Riemann mapping theorem

Applications to differential equations; for example:
Harmonic functions and DIrichlet problem
Conformal mappings and their use in solving the Dirichlet problem
Asymptotic evaluation of integrals, Bessel functions

Applications to signal processing; for example:
Fourier transforms, Laplace transforms, Paley-Weiner theorems, analytic signals

Applications to algebraic geometry, arithmetic geometry, and number theory; for example
Riemann surfaces, covering spaces, monodromy, winding numbers
Elliptic functions and complex tori
Riemann zeta function and prime number theorem

Recommended Texts:

Complex variables, by Ablowitz & Fokas
Functions of One Complex Variable, by Conway,
Complex Analysis, by Stein & Shakarchi.
Complex Variables, by Ash & Novinger (free online)
Applied Complex Variables, by Dettmann (Dover)
Complex Analysis, by Ahlfors
Functions of a Complex Variable, by Carrier, Crook, & Pearson

Math 545 - Partial Differential Equations I

M545 assumed knowledge: multivariable calculus, basic understanding of ordinary differential equations and mathematical models at the undergraduate level.

Recommended topics:
Derivation of the heat equations as a model for diffusion.
The Laplace and Poisson equations as steady-state limits.
Solution of the Laplace and heat equations by separation of variables.
Basics of Fourier analysis.
Maximum and minimum principles.
Derivation of the wave equation as a model for second-order hyperbolic equations that result from Newtonian mechanics.
Solution of the wave equation by separation of variables.
Solution of the wave equation via d'Alembert's solution.
Classification of second order equations as elliptic, parabolic, and hyperbolic.
Derivation of the advection equation as a model for first-order hyperbolic equations.
Solution of first-order equations via characteristics.
Nonlinear conservation laws.
Quasilinear equations, jump condition and propagation of shock waves.
Introduction to distributions, fundamental solutions, and Green's functions.

Optional topics: applications to biomedical engineering. MATH 545 is one of the courses taken by graduate students in the School of Biomedical Engineering.

Recommended Texts:
Partial Differential Equations of Mathematical Physics and Integral Equations, Guenther - Lee;
Partial Differential Equations of Applied Mathematics, Zauderer;
First Course in PDEs with Complex Variables and Transform Methods, Weinberger.
Introduction to Partial Differential Equations with Applications, Zachmanoglou
PDEs, an introduction, by Strauss

Math 560 - Linear Algebra

M560 assumed knowledge:
Finite dimensional real vector spaces, linear combinations, linear independence, span, basis.
Matrices, row/column reduction, solution of finite systems of linear equations.

Recommended topics:

1. Foundations (~2 weeks)
   - Vector spaces, subspaces, linear combinations
     (infinite dimensional and field independent descriptions)
   - Possible examples to encourage breadth: function spaces, polynomial and power series spaces, code spaces (or other finite field examples).

2. Geometry (~2 weeks)
   - Norms both finite and infinite (e.g., L^p spaces)
   - Inner products, least square
   - Orthonormal basis and projections (e.g., by Gram-Schmidt)
   - Isometries (Orthogonal, unitary)

3. Homogeneous Spectral Theory (~4 weeks)
   - Eigenvalues, Eigenspaces
   - Determinants, Trace
   - Algorithms to compute (minimal polynomial/characteristic)
   - Generalized Eigenspaces
   - Jordan Normal Form

4. Heterogeneous Spectral Theory (~2 weeks)
   - Singular Values, Singular Vectors
   - Principal Components

Optional topics:
For topic 1:
Affine/projective geometry
Hamel basis vs. infinite linear combinations

For topic 2:
Affine/projective geometry
Linear Automorphisms: general linear, unitary,
Adjoint operators: symmetric/Hermitian

For topic 3:
Field extensions
Spectra applications in Graph Theory / Probability / Stats

For topic 4:
SVD in general fields (intro Grassmannian)

Additional:
Matrix decompositions, e.g., LU, LUP, QR.
Tensors, exterior differential forms, p-vectors and inner product spaces.
Stochastic matrix, Markov chain, graph Laplacian.
Coding Theory (generator matrix, dual code)

Related topics covered in other classes:
Existence/Uniqueness of Finite Fields (567)
Krylov Methods (561).
Numerical Stability / Approximation (561)

Recommended Textbooks:
Linear Algebra done right, by Axler
Matrix Analysis, R. Horn and C. Johnson (for Jordan canonical form)
Linear Algebra, by P. Lax, John Wiley and Sons
Advanced Linear Algebra, by Roman

Math 566: Abstract Algebra I

M566 assumed knowledge: undergraduate level experience with groups, rings, fields, and vector spaces; mathematical reasoning and proof techniques, modular arithmetic and Euclidean algorithm.

The M566 course will include both abstract algebraic principles and explicit examples.

Recommended topics:
Groups:
Examples of groups: cyclic, dihedral, symmetric, alternating,
Matrix groups GL(n), SL(n).
Structure of groups: subgroups, center, order of elements, Lagrange's theorem.
Generators, relations, free groups, direct products.
Homomorphisms, Isomorphisms, Automorphisms, conjugation, kernel, image.
Quotient groups: cosets, normal subgroups, quotient groups, first isomorphism theorem.
Group actions: orbits, stabilizers, fundamental theorem of permutation representation.
Cayley's Theorem, Sylow Theorems I and II.
Solvable and simple groups.

Rings:
Examples of rings: Z/N, polynomial rings and factorization, Gaussians, quaternions
Matrix rings
Structure of rings: subrings, center, units, zero-divisors, fields.
Homomorphisms, Isomorphisms, Endomorphisms.
Ideals: cosets, ideals, quotient rings, prime ideals, maximal ideals.
Principal ideal domains, unique factorization domains.

Optional topics:
Optional topics for groups:
PGL, PSL, O(n), SO(n)
Sylow Theorem part III and classification of groups of small order.
Semi-direct products, short exact sequences.
Universal properties, category theory, free products.
Central series, p-groups.
Vector spaces, groups associated with bilinear forms (also covered in M567).

Optional topics for rings:
Sun Ze Chinese Remainder Theorem (also covered in M567), idempotents.
Euclidean domains, Noetherian domains.
Irreducibility criteria, elementary symmetric polynomials.
Group rings.

Recommended Textbooks:
Abstract Algebra, by Dummit and Foote
Algebra, by Artin
Algebra: a Graduate Course, by Isaacs
Advanced Modern Algebra, by Rotman
Algebra by Hungerford
Abstract Algebra: the basic graduate year (on-line), by Ash

Math 570 - Topology I

M570 Assumed Knowledge:
Basic understanding of these topics at the undergraduate level: metric spaces, sequences and convergence, open and closed sets, continuous functions, and topics in abstract algebra such as groups, generators and relations, and homomorphisms.

Recommended topics
Topological spaces, open sets, continuity, connectedness, compactness.
Homotopy and homotopy equivalence.
Fundamental group, covering spaces.
Homology (at least one of simplicial, singular, and CW).

The faculty member teaching the course has the choice to emphasize the foundations of topology (e.g., subspaces, product spaces, quotient spaces, Hausdorffness, compactness) or to emphasize the theory of fundamental groups and homology
(e.g., Van Kampen theorem, universal covers, other types of homology).

Optional topics:

Topological manifolds, Whitney embedding theorem.

Smooth manifolds, differential forms.

Long exact sequence of a pair, excision, Mayer-Vietoris.

Recommended Textbooks:
Introduction to Topological Manifolds, by Lee
Topology, by Munkres
Elements of Algebraic Topology, by Munkres
Algebraic Topology, by Hatcher

Math 574: Introduction to Mathematics Education Research

Math 574 assumed knowledge:
An understanding of how to read and discuss scholarly articles, write a thesis statement and provide supporting evidence, and proof-read/edit papers. Basic understanding of statistical methods.

Course Learning Objectives:
Upon successful completion of this course students will be able to:

1. summarize and synthesize past research that has influenced the direction of mathematics education today and identify promising areas of research for the promotion of the field,
2. describe and pursue applications of research for classroom practice,
3. identify a variety of learning theories relevant to mathematics education research,
4. critically design, evaluate, and compare and contrast the nature and applications of a variety of designs in mathematics education research,
5. prepare a research proposal on a related mathematics education research, and
6. describe and follow ethical guidelines in mathematics education research

Recommended discussion topics

- purpose of research studies,
- structure of literature reviews,
- importance of theory and theoretical perspectives and frameworks,
- ethical considerations,
- various research designs and methods (e.g., quantitative, qualitative and mixed research methods, design experiments, teaching experiments, action research, …), and
- writing an introduction, purpose statement, research questions, literature review, and research methods as part of a research proposal.

Recommended Textbooks:
Classics in mathematics education research. National Council of Teachers of Mathematics
by editors Carpenter, Dossey, & Koehler
Research design: Qualitative, quantitative, and mixed methods approaches, by Creswell