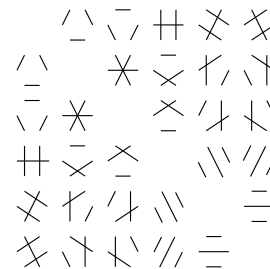# Mathematics Seminar

Rocky Mountain Algebraic Combinatorics Seminar

A Potpourri of Projective 2-Designs

Emily King
Colorado State University

Zauner conjectured that for all d there exists a set of d^2 equiangular vectors in C^d (SICs or Gabor equiangular tight frames). Another important open problem concerns the existence of $d + 1$ so-called mutually unbiased bases (MUBs) in C^d (maximal sets of MUBs) for any d not equal to a prime power. Both classes of objects are complex projective 2-designs. In this talk, the basics of (weighted) projective 2-designs will be discussed, including their recent generalization to finite fields. A collection of new results will be presented.

Pandey, Paulsen, Prakash, and Rahaman recently proposed an approach to make quantitative progress on Zauner's Conjecture in terms of the entanglement breaking rank of a certain quantum channel. This quantity is equal to the size of the smallest weighted projective 2-design. A construction of an infinite class of projective 2-designs over finite fields will also be presented, time permitting. This construction makes use of difference sets. Finally, in the quaternionic setting, every tight projective 2-design for H^d determines an equi-isoclinic tight fusion frame of $d(2d-1)$ subspaces of $R^{d(2d+1)}$ of dimension 3. This is joint work with Joey Iverson and Dustin Mixon.

Representation theoretic Hopf algebras and q-chromatic symmetric functions

Lucas Gagnon
UC Boulder

Hopf algebras are a nice way to incorporate algebraic tools into the study of combinatorial objects with recursive structure, and in some cases these combinatorial Hopf structures also encode representation theoretic information. This talk will use a new example to explain this general idea. On the combinatorial side is the chromatic quasisymmetric function, a q-analogue of Stanley's chromatic symmetric function of a graph; this function can be realized as the image of a homomorphism between Hopf algebra structures on graphs and (quasi-)symmetric functions. On the representation theory side is a collection of $GL_n(F_q)$-modules that arise in the study of the maximal unipotent upper triangular subgroup of $GL_n(F_q)$. I will show that the $GL_n(F_q)$-modules are, up to a canonical equivalence, a representation-theoretic realization of some chromatic quasisymmetric functions. Along the way I will define a new Hopf algebra of class functions on the unipotent upper triangular groups and use lots of pictures and examples.

Weber 223
4–6 pm, Friday, May 6, 2022
(Refreshments 3:30–4 pm)
Colorado State University
4 pm, Friday, May 6, 2022

This is a joint Denver U / UC Boulder / U of Wyoming / CSU seminar that meets biweekly.
Anyone interested is welcome to join us at a local restaurant for dinner after the talks.

Colorado State University

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