

QE Syllabi

Notes:

The words "final examination" always mean the two-hour, in-class, final examination for the course. Thus, in the following, any references to "final examinations" do NOT include any take-home parts of a final examination that are used by an individual instructor to determine a student's course grade.

These Syllabi are those emerging from the various Syllabi Committees formed in late Fall 2009, and were approved by faculty ballot on May 10, 2010.

MATH 501 - Combinatorics I

The core set of topics for MATH 501 are:

1. Basic counting: binomial coefficients, pigeonhole principle, counting with or without repeats, with or without order; double count; power set; binomial theorem; inclusion-exclusion.
2. Generating functions, recursion, recurrence relations and how to solve them; famous sequences.
3. Advanced counting: inversion techniques, groups and actions, orbits.
4. Equivalence relations, partitions, partially ordered sets, special orderings.
5. Projective planes over finite fields.
6. Graphs and networks (Euler, Hamilton, matchings, optimization).

The MATH 501 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Optional topics:

The student may expect 1-3 weeks to be devoted to several, but not all, of the following optional topics. These topics will not be covered on the final examination for MATH 501.

1. Logic and set theory.
2. Automata.
3. Algorithms.
4. Structures in projective planes.
5. Special classes of graphs (in particular strongly regular graphs).
6. Structural theory of graphs (isomorphism and reconstruction).
7. Counting graphs using Burnside / Cauchy / Frobenius; Polya theory.
8. The twelvefold way and extensions (as in Richard P. Stanley's text Enumerative Combinatorics, Volume I).
9. Coding theory and the Hamming graph, code constructions, code bounds, Paley graphs, posets and lattices.

10. Stirling numbers.
11. Moebius function.
12. Latin squares, MOLS.

Texts:

Sample Texts: Peter Cameron: Combinatorics; Jonathan Gross: Combinatorial Methods

MATH 502 - Combinatorics II

The core set of topics for MATH 502 include Designs, Graphs, Codes, Geometries and their links:

1. Designs: basic theory - parameters, Fisher's inequality, symmetric designs, incidence matrices, groups of designs.
2. Codes: Hamming distance, rate, block codes, length, error-correction, error-detection, linear codes, generating matrix, parity-check matrix, perfect codes, groups of codes.
3. Graphs: regularity, distance, diameter, girth, bipartite, covers, adjacency matrices, groups of graphs.
4. Geometries: affine and projective spaces and their groups.

The MATH 502 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Optional topics:

The student may expect 1-3 weeks to be devoted to several, but not all, of the following optional topics. These topics will not be covered on the final examination for MATH 502.

Symmetric functions, association schemes, distance regular graphs, extremal combinatorics, combinatorial algorithms.

Texts:

Sample Texts: van Lint and Wilson: Combinatorics; Cameron and van Lint: Designs, Graphs, Codes and their links; Pless: Coding theory; Taylor: The geometry of the classical groups; van Lint: Introduction to coding theory; Ling and Xing: Coding theory; Cameron's web notes on projective and polar spaces.

MATH 510 - Linear Programming and Network Flows

The core set of topics for MATH 510 are:

1. Introduction to optimization and problem formulation.
2. The geometry of linear programs.

3. The Simplex method.
4. Duality theory.
5. Sensitivity analysis.
6. Network Flows.
7. Interior point methods.
8. Integer programming.

The MATH 510 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Optional topics:

The student may expect 1-3 weeks to be devoted to several, but not all, of the following optional topics. These topics will not be covered on the final examination for MATH 510.

1. Game theory.
2. Portfolio selection.
3. Quadratic programming.
4. Convex optimization.
5. Applications, e.g., to Engineering and Agriculture.
6. Matlab Projects

Texts:

Sample Texts: Introduction to Linear Optimization, D. Bertsimas and J.N. Tsitsiklis.

MATH 517 - Introduction to Mathematical Analysis

The core set of topics for MATH 517 are:

1. Metric spaces, compactness, completeness.
2. Sequences, convergence, Cauchy sequences.
3. Series, power series, nonnegative and absolutely convergent series.
4. Continuity, uniform continuity, intermediate value theorem.
5. Sequences and series of functions, pointwise and uniform convergence.
6. Weierstrass approximation theorem, equicontinuity, the Arzela-Ascoli theorem.
7. Differentiation in several variables, partial derivatives, the chain rule.

8. Linearization, mean value theorems, sequences of differentiable functions.
9. Higher order derivatives, power series, Taylor's theorem.
10. Contraction mapping principle, implicit and inverse function theorems.

The MATH 517 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Optional topic:

This topic will be covered at the instructor's option, and will not be covered on the final examination for MATH 517.

1. Classification of critical points.

Text:

Rudin: Principles of Modern Analysis

MATH 519 - Complex Analysis

The core set of topics for MATH 519, covering a bit less than 12 weeks of the course, are:

2. Functions on the complex plane: convergence, continuous functions, holomorphic functions, power series, integration along curves, Cauchy-Riemann equations.
3. Cauchy's Theorem and its Applications: Goursat's theorem, local existence of primitives, Cauchy's integral formulas, Morera's theorem, sequences of holomorphic functions, holomorphic functions in terms of integrals, Schwarz reflection principle, Runge Approximation, Liouville theorem, Maximum modulus.
4. Meromorphic functions and the logarithm: The residue formula, singularities and meromorphic functions, the argument principle, homotopies and simply connected domains, the complex logarithm, fourier series and harmonic functions, Rouché's theorem, Techniques of integration.
5. Entire functions: Jensen's formula, functions of finite order, infinite products, Weierstrass infinite products, Hadamard factorization.
6. Conformal mappings: conformal equivalence, Dirichlet problem, Schwarz lemma, automorphisms of disc, Riemann mapping theorem.

The MATH 519 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Optional topics:

The student may expect up to a bit more than 3 weeks to be devoted to several, but not all, of the following optional topics. These topics will not be covered on the final examination for MATH 519.

1. Asymptotic evaluation of integrals
2. Bessel functions, Stirling's formula, Poisson integral formula

3. Conformal mappings onto polygons, Schwarz Christoffel integral
4. Covering spaces, monodromy
5. Elliptic functions, complex tori
6. Fourier transform, Paley-Weiner theorem and applications to differential equations
7. Gamma functions* and their analytic continuation
8. Generalized Cauchy integral formula
9. Ideal fluid flow
10. Laplace transform and applications to differential equations
11. Minimal surfaces
12. Mittag-Leffler theorem
13. Modular character of elliptic functions, Eisenstein series
14. Modular functions
15. Riemann Hilbert problems
16. Riemann zeta function*, Prime number theorem
17. Sheaf of germs of holomorphic functions
18. Theta functions, two and four squares theorem

*: Instructors are particularly encouraged to cover the Riemann zeta function and gamma functions if time permits.

Texts:

Suggested textbooks (alphabetical by author): Complex variables by Ablowitz & Fokas, Functions of One Complex Variable by Conway, Complex Analysis by Stein & Shakarchi.

MATH 540 - Dynamical Systems and Chaos

The core set of topics for MATH 540 are:

1. Systems of Differential Equations: (Review) existence and uniqueness of solutions, (review) continuous dependence of solutions on initial conditions and parameters.
2. Autonomous Systems: Phase space, vector fields, orbits and flows; critical points and equilibrium solutions; linearization; periodic solutions; first integrals and integral manifolds; Liouville's theorem.
3. Critical Points: Linear systems: eigenvalues and diagonalization; classification of 2D linear systems; remarks on classification of 3D linear systems; critical points of nonlinear systems; review stable and unstable manifolds.

4. Periodic Solutions: Periodic solutions of 2D systems; Bendixson criterion; Poincaré-Bendixson theorem and applications; existence of periodic solutions in higher-dimensional systems.
5. Introduction to Stability Theory: Examples; stability of equilibrium solutions; stability of periodic solutions; linearization.
6. Linear Systems: Fundamental matrices; systems with constant coefficients; nonautonomous linear systems; systems with periodic coefficients.
7. Stability Theory Continued: Stability of equilibrium solutions by linearization; stability of periodic solutions of autonomous equations; Lyapunov functions; Hamiltonian systems and systems with first integrals.
8. Introduction to Perturbation Theory: Examples; order functions and time scales; Poincaré expansion theorem.
9. Poincaré-Lindstedt Method: Periodic solutions of autonomous 2nd order equations; approximation of periodic solutions on arbitrary times scales; periodic solutions of equations with forcing terms; existence of periodic solutions.
10. Method of Averaging: Lagrange standard form; averaging in the periodic case; averaging in the general case.
11. Relaxation Oscillations: Mechanical systems with large friction; Van der Pol equation; Lotka-Volterra equations.
12. Bifurcation Theory: Poincaré normal forms; averaging and normal forms; center manifolds; bifurcation of equilibrium solutions (saddle-node, transcritical, and pitchfork bifurcation); Hopf bifurcation.
13. Introduction to Iterated Maps: Poincaré maps; stability and bifurcation of periodic orbits.

The MATH 540 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Optional topics:

The student may expect that about three weeks will be devoted to a selection from the following optional topics. These topics will not be covered on the final examination for MATH 540.

1. Chaotic Dynamics: Lorenz system and Lorenz maps; chaotic 1D maps: quadratic map and tent map; Sharkovsky theorem and Feigenbaum numbers; fractal sets: limit capacity and Hausdorff dimension; correlation dimension, information dimension, dimension spectrum; Lyapunov exponents.
2. Review of other aspects of chaotic dynamics: Smale horseshoe and symbolic dynamics; homoclinic orbits and Melnikov method; Julia sets; Shilnikov bifurcation; time series embedding; shadowing.
3. Applications of dynamical systems (from supplementary texts).
4. Continued discussion of the method of averaging (adiabatic invariants, resonance manifolds).
5. Continued discussion of bifurcations and normal forms, including codimension-two bifurcations.
6. Hamiltonian systems.
7. Introduction to delay differential equations.

8. Introduction to symbolic dynamics.
9. Chaotic time series analysis and numerical experiments.
10. Coupled cell systems and neural networks.

Texts:

Suggested Textbook: F. Verhulst: Nonlinear Differential Equations and Dynamical Systems, Springer 2006 (3rd edition).

Supplementary texts: S. Strogatz: Nonlinear Dynamics and Chaos, Addison Wesley 1994; M. Kot: Elements of Mathematical Ecology, Cambridge University Press 2001; J.D. Murray: Mathematical Biology, Springer 1989.

MATH 545 - Partial Differential Equations I

The core set of topics for MATH 545 are:

1. Classification of PDEs.
2. Conservation laws.
3. Characteristics.
4. Quasilinear equations, jump condition and propagation of shock waves.
5. Hyperbolic equations: the 1-D wave equation, D'Alembert's solution, method of characteristic parallelograms.
6. Separation of variables.
7. Fundamental solutions and Green's functions.
8. Elliptic Equations: Laplace's equation.
9. Parabolic Equations: Diffusion equation.
10. Maximum and minimum principles.

The MATH 545 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Optional Topics:

MATH 545 is also a core course for graduate students in the School of Biomedical Engineering. Thus, the course must contain a significant number of biomedical examples. The choice of examples may vary depending upon the instructor, and therefore will not be part of the final examination.

Texts:

Suggested texts: Partial Differential Equations of Mathematical Physics and Integral Equations {Guenther and Lee; Partial Differential Equations of Applied Mathematics {Erich Zauderer; A First Course in Partial Differential Equations with Complex Variables and Transform Methods { H.L. Weinberger.

MATH 546 - Partial Differential Equations II

The core set of topics for MATH 546 are:

1. Variational formulation of PDEs, weak solutions, test functions
2. Weak derivatives, the Sobolev spaces $W^{k,p}$.
3. Sobolev embedding theorems.
4. The extension theorem, the trace theorem.
5. Weak solutions of second order elliptic equations, elliptic regularity.
6. Weak solutions of parabolic equations, energy estimates, maximum principles.
7. Distributions.
8. Fundamental solutions and Green's functions, application to nonhomogenous problems
9. Fourier transforms.
10. Viscosity and entropy solutions of hyperbolic equations.

The MATH 546 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Texts:

Suggested texts: An Introduction to Partial Differential Equations {Renardy and Rogers; Partial Differential Equations {Evans; Partial Differential Equations: Methods and Applications {McOwen; Introduction to Partial Differential Equations {Folland.

MATH 560 - Linear Algebra

The core set of topics for MATH 560 are:

1. Fundamental Concepts: Vector spaces, subspaces; dependence, span, basis, extension to form a basis; dimension, direct sums; congruence.
2. Duality: Linear functions; annihilators, co-dimension.
3. Linear Transformations: Definition, range, nullspace; algebra of transformations; invertible maps; adjoint operators; annihilators; similarity transformations; projections.

4. Matrices: Bases and representations; vector-wise and block-wise interpretations; composition; rank; adjoint; special matrices; change of bases.
5. Determinant and Trace: Multilinear functions, volume; properties of determinants; Cramers Rule.
6. Spectral Theory: Iterated maps, power methods; eigenvalues, eigenvectors, characteristic polynomials; distinct eigenvalues and independence; Spectral Mapping Theorem; Cayley-Hamilton Theorem; similarity, minimal polynomials; spectral decomposition; diagonalizable matrices; adjoint and commuting matrices, multiplicity.
7. Euclidean Spaces: Inner products, norms, inequalities; orthonormal bases, Gram-Schmidt procedure; representation theorem; orthogonal complements, decompositions; projections, distance; adjoint operators; norm of an operator; isometries, orthogonal maps.
8. Normed Linear Spaces: Norms, distance, balls, P-norms, Holder inequality, equivalence; bounded operators; dual norm, continuity of invertibility; norms of matrices.
9. Unitary Equivalence and Normal Matrices: Unitary matrices, unitary equivalence; Schur decomposition; normal matrices; QR factorization.
10. Jordan Canonical Form
11. Singular Value Decomposition: Polar Decomposition; Singular Value Decomposition; generalized Inverse.

The MATH 560 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Optional topics:

The student may expect that up to three weeks will be devoted to various optional topics such as spectral theory for self adjoint maps, special matrices such as Toeplitz and circular, and/or PLU decomposition. These topics will not be covered on the final examination for MATH 560.

Texts:

Suggested Textbooks: Linear Algebra, P. Lax, John Wiley and Sons, Inc., 2nd Edition, ISBN 0-471-75156-4; Matrix Analysis, R. Horn and C. Johnson, Cambridge University Press, ISBN 0-521-38632-2 (for Jordan canonical form).

MATH 561 - Numerical Analysis I

The core set of topics for MATH 561 (a course in numerical linear algebra) are:

1. Vector and matrix norms, sparse matrix representations, floating number arithmetic.
2. Condition numbers, stability analysis.
3. Gaussian elimination (GE) with partial pivoting GE applied to digonally dominant, tridiagonal, and banded systems.
4. SPD matrices, Cholesky (CHOL) factorization.
5. Basic iterative schemes and convergence conditions Jacobi, Gauss-Seidel (GS), and SOR iterative schemes.

6. Conjugate gradient (CG) methods.
7. Singular value decomposition (SVD).
8. Least squares problems.
9. Householder reflections and QR factorization.
10. Eigenvalues: QR iterations without and with shifts.
11. Krylov subspace methods: Arnoldi iteration and GMRES.
12. Preconditioning.

The MATH 561 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Optional topics and applications:

The following optional topics can be covered at the instructor's discretion, but will not be covered on the final examination:

1. Tensor products of matrices.
2. Red-black ordering in iterative schemes.
3. Eigenvalues: power methods.
4. Krylov subspace methods: BiCGStab.
5. Krylov subspace methods: Lanczos iteration.

The following optional applications may be covered to enhance students' understanding of the course materials but will not be covered on the final examination:

1. GE: Finite differences for ODE boundary value problems.
2. CHOL: L^2 orthogonal projection into polynomial subspaces.
3. GS: 2-dim Poisson equation boundary value problem.
4. GMRES: Taylor-Hood Q2Q1 finite elements for 2-dim Stokes flow.
5. SVD: Image compression and data compression.

Texts:

Suggested Textbooks: [1] Demmel, "Applied Numerical Linear Algebra", SIAM, 1997; [2] Trefethen, Bau, "Numerical Linear Algebra", SIAM, 1997.

MATH 566 - Abstract Algebra I

The core set of topics for MATH 566 are:

- I. Groups:
 - a) Basic examples, subgroups, cosets, homomorphisms. Note that lots of examples will be given throughout lectures/assignments in all weeks.
 - b) Group Actions.
 - c) Sylow theorems and applications.
 - d) Direct and semi-direct products.
- II. Rings:
 - a) Basic examples, subrings, ideals, homomorphisms.
 - b) Factorization in integral domains.

The MATH 566 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics; examples discussed in class or homework may also be covered on that examination. Students may also be expected to discuss new examples, in the context of the concepts above, on the final examination.

Texts:

Suggested Texts: Dummit and Foote, Abstract Algebra; Hungerford, Algebra; Artin, Modern Algebra. These texts are typically used for both 566 and 567.

MATH 567 - Abstract Algebra II

The core set of topics for MATH 567 are:

- I. Modules:
 - a) Basic definitions and examples, quotient modules, homomorphisms of modules. Note that lots of examples will be given throughout lectures/assignments in all weeks.
 - b) Module generation and direct sums.
 - c) Finitely generated modules over PIDs and applications.
- II. Fields:
 - a) Field extensions and algebraic extensions.
 - b) Splitting fields and separability.
 - c) Galois theory and applications.

The MATH 567 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics; examples discussed in class or homework may also be covered on that examination. Students may also be expected to discuss new examples, in the context of the concepts above, on the final examination.

Texts:

Suggested Texts: Dummit and Foote, Abstract Algebra; Hungerford, Algebra; Artin, Modern Algebra. These texts are typically used for both 566 and 567.

MATH 570 - Topology I

The core set of topics for MATH 570 are:

1. Topological spaces, bases and subbases of topologies, metric spaces and metric topologies, continuous functions, subspace topologies, connectedness, compactness (lots of examples given throughout lectures/assignments in all weeks).
2. Function spaces, product spaces, the Tychonoff theorem (full proof not given, but students must understand the product topology for an arbitrary product). Topological groups.
3. Separation axioms. Brief mention of Urysohn's lemma, Tietze Extension Theorem (full proofs not necessarily given), path connectedness.
4. Quotient spaces.
5. Homotopy: general definition, path homotopy, contractibility, the fundamental group, homotopy equivalence. More on quotient spaces {cones, suspensions, joins}.
6. Basics of covering space theory.

The MATH 570 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics; examples discussed in class or homework may also be covered on that examination. Students may also be expected to discuss new examples, in the context of the concepts above, on the final examination.

Texts:

Sample Texts: Topology, Munkres; Topology, Hocking and Young; An Introduction to Algebraic Topology, J.J. Rotman. There are many other suitable texts.

MATH 617 - Integration and Measure Theory

The core set of topics for MATH 617 are:

1. Set theory, sigma-rings and sigma-algebras
2. Finitely additive measures, outer measure and measurable sets
3. Lebesgue measure in one dimension, Borel measures
4. Simple functions, measurable functions, Lebesgue integration
5. Bounded and dominated convergence theorems

6. Product measures
7. Fubini's theorem
8. The Radon-Nikodym theorem
9. Lebesgue measure and integration in finite dimensions
10. Change of variables in finite dimensions

The MATH 617 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Optional topics:

These topics will be covered at the instructor's option, and will not be covered on the final examination for MATH 617.

1. Riemann Integration.
2. Null sets and Lebesgue's characterization of Riemann integrability.
3. L_p spaces.
4. Probability spaces, independence, law of large numbers, central limit theorem.

Text:

Inder K. Rana - An Introduction to measure and integration

MATH 618 - Advanced Real Analysis

The core set of topics for MATH 618 are:

1. Banach spaces, L_p spaces.
2. Linear operators and linear functionals.
3. Hahn-Banach theorem.
4. Dual spaces.
5. Hilbert spaces, L_2 spaces.
6. Fourier analysis.
7. Linear operators and solution of linear inverse problems: adjoint, Baire's theorem, open mapping theorem, uniform boundedness principle, closed graph theorem, Neumann perturbation theorem.

The MATH 618 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Optional topics

These topics will be included at the instructor's option, and will not be covered on the final examination for MATH 618.

1. Fixed point theory; Banach, Brouwer, Schauder, and Kakutani fixed point theorems.
2. Calculus for nonlinear operators.

Texts:

Real Analysis, Folland; Principles of Functional Analysis, Schechter; Handbook of Analysis and its Foundations; Analysis for Applied Mathematics, Cheney.

MATH 640 - Ordinary Differential Equations

The core set of topics for MATH 640 are:

1. Introduction: Examples of real-world models as differential equations { population dynamics, mechanical systems, electrical circuits, fluid flow; one-dimensional dynamics { phase line, separation of variables; two-dimensional dynamics { introduction to phase plane; three-dimensional dynamics { a glance at possibility of chaotic dynamics.
2. Linear Systems: Matrix ODEs; eigenvalues and eigenvectors, diagonalizability; classification of 2D linear systems; exponentials of linear operators; fundamental solution theorem; complex and multiple eigenvalues; semisimple nilpotent decomposition and matrix exponential; matrix exponential via Cayley Hamilton theorem; linear stability; non-autonomous linear systems and Floquet theory.
3. Existence and Uniqueness of Solutions: set and topological preliminaries in \mathbb{R}^n -convergence and uniform convergence; function space preliminaries { metric spaces, contraction maps, Lipschitz functions; existence and uniqueness theorems; Gronwall inequality, continuous dependence on initial conditions and parameters; maximal interval of existence.
4. Dynamical Systems: Definitions { deterministic dynamical systems, orbits, invariant sets; flows and vector fields; global existence of solutions of first order systems; equilibrium points and linearization, hyperbolic and non-hyperbolic equilibria; stability of equilibria, Lyapunov functions; topological conjugacy and equivalence; Hartman-Grobman theorem; limit sets, attractors, basin of attraction; stability of periodic orbits, Poincaré maps.
5. Invariant Manifolds: Stable and unstable sets; homoclinic and heteroclinic orbits; stable manifolds; local stable manifold theorem; global stable manifolds; center manifold theorem and applications thereof.
6. Phase Plane: Nonhyperbolic equilibria in the plane { Two zero eigenvalues and non-hyperbolic nodes, imaginary eigenvalues, topological centers; symmetry and reversible systems; index theory in 2D, degree theory in higher dimensions; Poincaré-Bendixson theorem; behavior at infinity { Poincaré sphere.

The MATH 640 final examination (which may serve as a Qualifying Examination) will cover a selection of the above topics.

Optional topics:

The student may expect that about two weeks will be devoted to topics chosen by the instructor. These topics will not be covered on the final examination for MATH 640. Sample topics are:

1. Chaotic dynamics.
2. Bifurcation theory.
3. Hamiltonian dynamics.

Texts:

Suggested Textbook: One of J. Meiss: *Differential Dynamical Systems*, SIAM, 2007; L. Perko: *Differential Equations and Dynamical Systems*, Springer, 2001 (3rd ed.).