

Hackin' Hackenbush

Big Concepts:

- Graph Theory This game involves graphs, and will have students working with vertices and edges, as well as connected graphs.
- Game Theory This lesson looks at perfect games and winning strategies, specifically 2nd-player-winning games

Materials:

- red, blue, and black expo markers
- various colored toothpicks, separated by color
(need at minimum 2 different colored sets of 10 per pair of students)
- marshmallows
(need approximately 20 per pair of students, optional)
- paper and pencil
(1 set per pair at minimum)

Prep Before Lesson:

- Sort toothpicks by color
- Place a handful of 2 colors of toothpicks at each table
- Draw figures from the end of this sheet on the board in red and blue

Hackenbush Lesson Plan:

:00–:15 *Motivation:* Prompt the students with the game Monopoly (or another super common game). Ask the students who has won at a game of Monopoly. Have a few students to share their strategies for winning. Then, ask who has won Monopoly EVERY time they have played (hands should be none or just one or two). Rule out any hands to luck, and explain that there is no winning strategy to Monopoly since it isn't a perfect game.

:15–:30 *Math Terms:* Define/discuss the following terms

- Combinatorics/combinatorial games
- Perfect Game
- Graph (NOT a euclidean graph!)
- Vertex/vertices
- Edge/edges
- Connected

:30–:45 *Hackenbush:* Explain the rules of 2-color Hackenbush. Draw an example graph and play a game with the students to model how it is played.

1. Decide who is red and who is blue.
2. Decide who plays first and who plays second.
3. Draw a graph with red and blue edges, with at least one edge connected to the ground.
4. On a player's turn, a player may remove any edge of their color. If the graph becomes disconnected, the component of the graph not connected to the ground is removed from the game.
5. Whoever makes the last move wins!

:45–1:30 *Main Activity*: Have the students work on Figures 1-3 to figure out a winning strategy (if there is one) to the following graphs. Should time permit, have students create their own Hackenbush graphs and share to the class. Periodically bring the class back together and discuss findings so far. Also, at some point discuss the four ways to win.

Winning strategies for each figure:

- Figure 1: WLOG, red removes the left “leg” as their first move, and the right leg as their second. This strategy is *independent* of blues moves.
 - * This graph has more blue edges than red, but it is the position of the red edges that gives player red an advantage.
- Figure 2: If red removes the bottom right “leg”, then blue can remove the other two legs WLOG. If red removes something else, blue needs to remove edges that aren’t legs first.
 - * This graph has more blue edges than red, and this is why player blue can win.
- Figure 3: Player 2 must mirror the move of player 1 on the opposite tower.
 - * This graph has the same amount of blue edges and red edges. The two towers are the same with the colors swapped, which allows player 2 to win since they can remove the last edge.

General ways to win:

- Player red wins, regardless of if they go first or second
- Player blue wins, regardless of if they go first or second
- Player 1 wins, regardless of if they are red or blue
- Player 2 wins, regardless of if they are red or blue

1:30–1:45 *Closing Remarks* Finish discussing the strategies for Figures 1-3, and any student presentations. Reveal to students that 2-color Hackenbush can NEVER be a player 1 winning board (given optimal play). Describe other variations of Hackenbush (1 or 3 color, more players, new dimensions, etc.) and encourage the students to explore in the future. **HAVE THE STUDENTS CLEAN UP**, and thank them for attending!

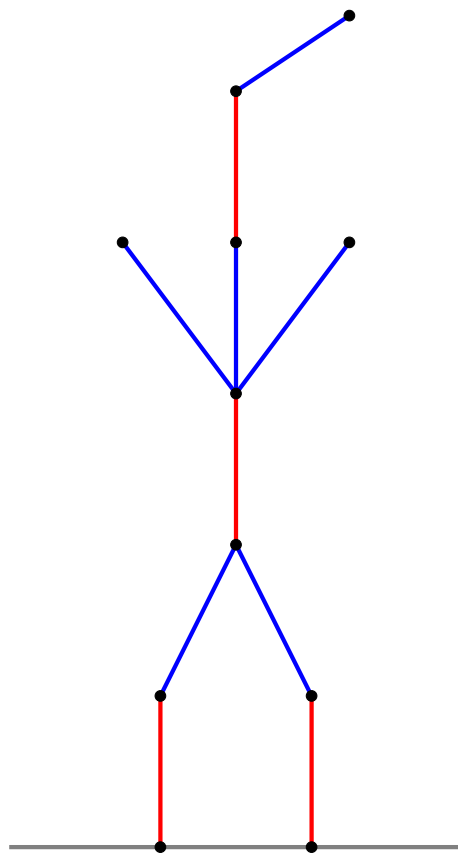


Figure 1: Red-winning

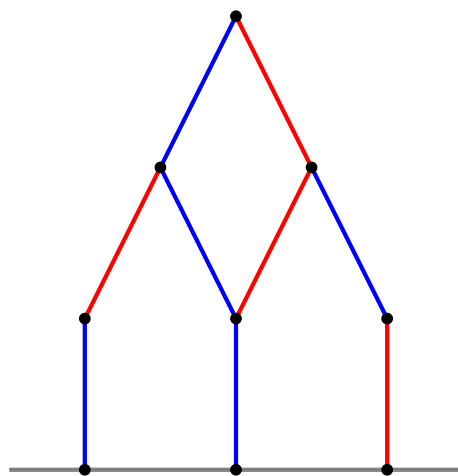


Figure 2: Blue-winning

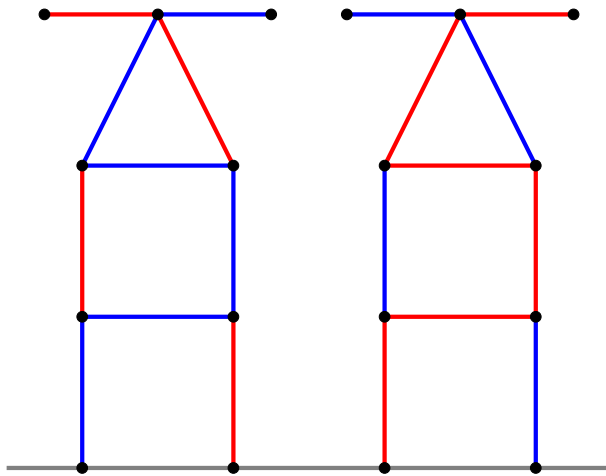


Figure 3: Player Two-winning